

*PACKING UNIT SPHERES INTO THE SMALLEST SPHERE  
USING VNS AND NLP*

Rym M'Hallah 1  
Department of Statistics and Operations Research  
Kuwait University  
Kuwait, Kuwait

**\*\*Abdulaziz Alkandari 2**  
**Applied Sciences Department**  
**Public Authority for Applied Education and Training**  
**Kuwait, Kuwait**  
**Email: abdl\_alkandari@hotmail.com**

Nenad Mladenovic 3  
Mathematical Sciences Department  
Brunel University  
Uxbridge, United Kingdom

**Abstract**

In this research, we present the optimization problem of packing identical spheres (PSS) of unit radii into the smallest sphere. It models PSS as a non-linear program (NLP) and approximately solves it using a hybrid heuristic which couples a variable neighborhood search (VNS) with a local search (LS). VNS investigates the neighborhood of a feasible local minimum  $u$  in search for the global minimum, where neighboring solutions are obtained by shaking one or more spheres of  $u$  and the size of the neighborhood is varied by changing the number of shaken spheres, the distance and the direction each sphere is moved. The computational investigation shows that the proposed hybrid heuristic provides more precise results than existing approaches.